Robust Rezoning in ALE Aided by Mesh Untangling

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Lagrangian methods are popular in the field of hydrodynamics. Such discretizations, in which the mesh moves with the fluid, have desirable features such as conservation of mass. However, in 2D and 3D when the simulated flow exhibits some rotation, the mesh can easily become tangled. In this case, the simulation fails. A common remedy to this entanglement problem is to introduce occasional mesh improvement (rezoning) followed by the remapping of physical quantities to the new and improved mesh. Such methods are referred to as arbitrary Lagrangian Eulerian (ALE) methods.

The rezone step in an ALE method typically relies on mesh smoothing which is perhaps triggered by some mesh quality indicators to keep the computational mesh in a valid state throughout a simulation. (An invalid mesh that has negative cell volumes results in simulation failure.) One drawback to this approach is that it is inherently heuristic and cannot guarantee a valid mesh throughout a simulation. In the case where, despite the rezoning step, a mesh becomes tangled, user intervention becomes necessary since conventional mesh smoothing algorithms typically cannot repair such a failed mesh.

We have developed a mesh untangling method that is based on the method presented in [1] and has improved robustness. The method is a purely geometric algorithm in 2D that is computationally efficient with a predictable cost per mesh vertex. Most importantly, it can be used not only as a mesh untangling method, but also as a mesh improvement method. In contrast to our method, most other existing robust mesh untangling algorithms are minimization-based and, thus, costly (for example, see [2]).

Our algorithm builds on two approaches to mesh untangling. The first approach is the feasible set method, in that a convex polygon, the feasible set, is constructed for each vertex in the mesh. This feasible set is the set of all coordinate positions that a vertex can occupy while maintaining mesh validity relative to its immediate neighbor cells. After the feasible set is constructed for a particular vertex in the mesh, this vertex then is placed at the centroid of the feasible set, resulting in an untangled mesh. The second approach is our weak untangling method. This method can be viewed as a discrete and, hence, efficient version of a more standard minimization-based untangling method. We use it to move severely tangled vertices, which cannot be untangled using the feasible set approach, closer to a configuration that is amenable to untangling by the feasible set method.

Figure 1 illustrates the construction of the feasible set for a patch of four cells whose center vertex is tangled. The feasible set method is the intersection of a number of half planes that are determined by the geometry of the patch surrounding the tangled vertex. For a patch of four quadrilaterals there are 12 half-plane intersections. The first image (A) in the sequence depicts the initial tangled mesh fragment. The second image (B) depicts the intersection of

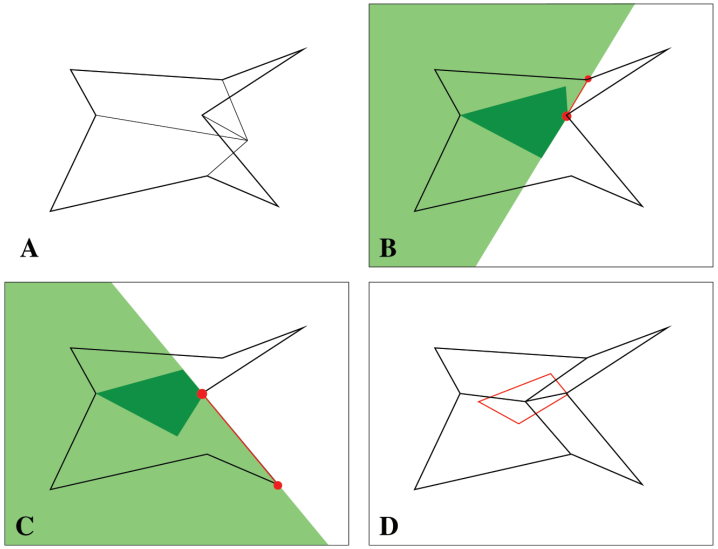

Fig. 1. The construction of the feasible set of a tangled vertex.

one of the half planes and the bounding box (light green) with the current feasible set (dark green). In the third image (C) we see another such half-plane intersection, and finally, image (D) depicts the feasible set in red with the previously tangled vertex placed at its centroid.

The feasible set untangling method alone can be used as a mesh smoothing method and, if used that way, is robust. Alternatively, it can be used in conjunction with a more traditional mesh smoothing method, such as Winslow's method, to repair the mesh when it tangles. Figure 2 depicts a snapshot of a simulation where a high pressure gas spills through a gap in a wall from the left to the right. In this example, our feasible set untangling method was used in tandem with standard Winslow mesh smoothing. Without mesh untangling the mesh becomes tangled at the corners of the wall gap soon after the start of the simulation, and the simulation fails. In contrast, with mesh untangling the simulation continues to run robustly.

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[1] P. Vachal, R.V. Garimella, M.J. Shashkov, *J. Comput. Phys.* **196,** 627-644 (2004).
[2] P. Knupp, L. Margolin, M. Shashkov, *J. Comput. Phys.* **176,** 93-128 (2002).

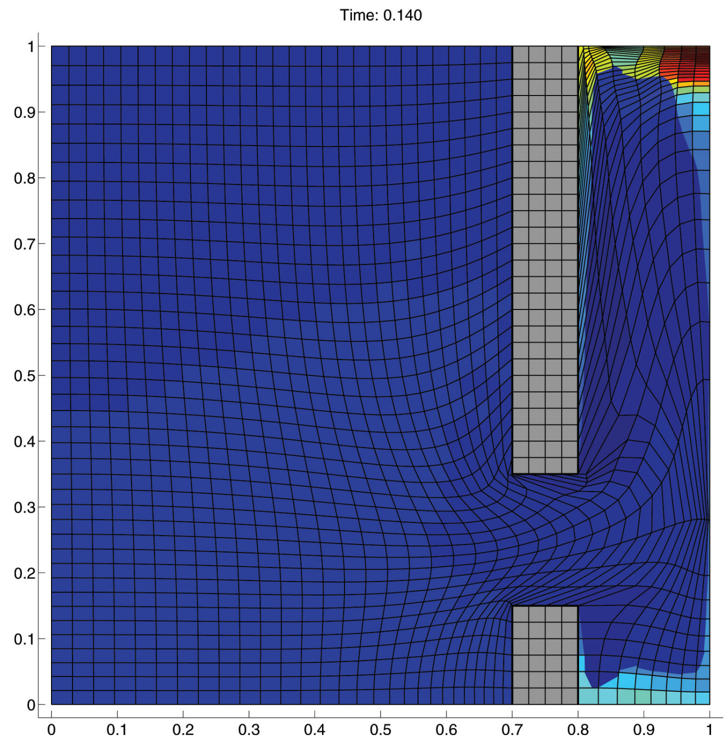

Fig. 2. Snapshot of an ALE simulation of high pressure gas on the left that spills through a hole in a wall. Feasible set untangling is employed in tandem with standard Winslow mesh smoothing during the rezone step. Without untangling, the simulation would fail at a much earlier time.

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